Analysis on the contribution of cross beam to a torsional buckling of thin, rectangular beam section

by Windu Partono

Submission date: 18-Oct-2017 04:55AM (UTC+0700)
Submission ID: 864437179
File name: am_to_a_torsional_buckling_of_thin,_rectangular_beam_section.pdf (299.18K)
Word count: 2500
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ANALYSIS ON THE CONTRIBUTION OF CROSS BEAM TO A TORSIONAL BUCKLING OF THIN, RECTANGULAR BEAM SECTION

(SC-015)

Sri Tudjono, Windu Partono, and Joko Purnomo

Department of Civil Engineering, Faculty of Engineering, Diponegoro University, Semarang, Indonesia
e-mail of corresponding author: tudjono@indosat.net.id, windu_partono@yahoo.com, a.joko.purnomo@gmail.com

ABSTRACT

In the analysis, a cross beam is idealized as a spiral spring having an equal rotational stiffness to the cross beam. Based on the torsional buckling exact solution, assuming a simply supported beam with a stiffened web subjected to a constant moment, the numerical solution of a torsional buckling is acquired. Moreover, the beam is assumed to have a constant segmental moment distribution. Based on basic equilibrium condition of the differential equation of each segment, constants of integration of the exact solution of the segmental rotations can be herein obtained. The number of constants of integration is twice the number of segments. On the basis of geometric boundary conditions at the beam's supports and at the joint nodes between segments, as well as natural boundary conditions of the segment joints equipped with spiral spring springs, the homogeneous equations as a function of constants of integration will be obtained. The determinant of the coefficient matrix of such homogeneous equation as a function of constants of integration is then evaluated. The determinant having a zero value identifies the critical torsional buckling moment. By decreasing the segmental length, the critical moment will convergence to the torsional buckling moment. It is proven that the analysis based on the segmental approach having a constant moment will closely match the exact solution. For a spiral spring having ratio of rotational stiffness to the beam's torsional stiffness equals to 1, the torsional buckling moment will closely approach that of the beam when it is supported at the location of the spiral spring. Moreover, for n equally distributed spiral springs, the torsional buckling moment will be (n+1) times the torsional buckling moment of an identical beam without spring. It is therefore concluded that the presence of Cross Beam can significantly increase the torsional buckling moment of a beam.

Keywords: Torsional buckling, cross beam, spiral spring.

1. INTRODUCTION

A thin rectangular beam is subjected to a constant moment (Mx) as shown in Figure 1. Due to its thin rectangular section, the beam will deform in three directions i.e. vertical (v), horizontal (u) and rotational (β) displacement. The beam is assumed to be under elastic material behavior following Hooke's law (small displacement) and when the beam buckles , the section is still in a rectangular shape while warping is not considered herein.

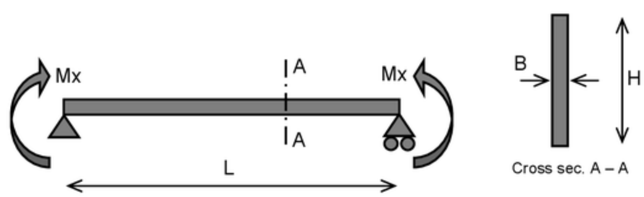

Figure 1. Thin rectangular beam subjected to constant moment

Based on elementary equilibrium, the governing differential equation can be derived as follows,

$$\frac{d^2\beta}{dz^2} + k^2\beta = 0 \text{ with } k^2 = \frac{Mx^2}{EI_yGJ} \quad \ldots\ldots\ldots(1)$$

with E refers to elastic modulus of material, G refers to shear modulus of material, I_y refers to moment inertia of beam's section through y-axis, J refers to torsional moment of inertia and k refers to a constant of equation. The general solution of Equation 1 is given as follows,

$$\beta = A\sin(kz) + B\cos(kz) \quad \text{.......(2)}$$

For a simply supported beam, it has boundary condition of β = 0 at z = 0 and z = L. By substituting such BC values to Eq.1, a critical moment formula can be obtained as given in Eq. 3.

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ} \quad \text{.......(3)}$$

In this paper, beam having cross beams that are idealized as spiral springs is studied. The cross beams idealized as spiral springs contribute to endure rotational displacement of the beam. The spiral spring's stiffness is assumed to be equal to cross beam'sbending stiffness. The spiral spring's and beam's stiffness are expressed as EI_z and GJ, respectively. Hereafter, the spiral spring's stiffness will be stated as C which is a ratio between spiral spring's stiffness EI_z and beam's stiffness GJ.

A number of spiral springs are added to the beam in a symmetrical condition with respect to its length. To study the effects of spiral springs to the enhancement of the beam's critical moment, the stiffness and the number of spiral springs are herein varied. The first mode shape of beam having one and two spiral springs are also shown hereafter.

The aforementioned equations given are also able to determine the critical moment of beam under a non-constant moment. In this study, a beam having a linear moment dsitribution is used to show the behavior of such non-constant moment. These non-constant moment can be discretized into a number of segments of a narrow width. Moment of each segment simply equals to the moment in the center of the segment. The critical moment of such condition can then be evaluated the same way by satisfying the determinant of the moment value to be equal to zero.

2. METHODS

Now we consider a simply supported beam having a spiral spring on its mid-length so as is shown in Figure 2. The beam is subjected to a constant moment throughout its length.

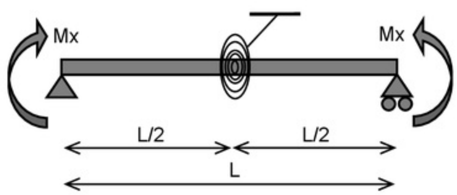

Figure 2. Beam having a spiral spring on its mid-length

As the beam is divided into two segments by the spiral spring, we consider the beam has two solutions as expressed in Equations 4. The solution which represents left part of beam is stated as β_1 and the one on the right is stated as β_2.

$$\beta_1 = A_1\sin(kz_1) + B_1\cos(kz_1)$$

$$\beta_2 = A_2\sin(kz_2) + B_2\cos(kz_2) \quad \text{.......(4)}$$

Notations A_1, A_2, B_1 and B_2 consequently refer to integration constants.

Due to the presence of spiral spring, additional 1 boundary condition and 1 natural geometry condition are herein obtained as given in Equations 5. The first boundary condition refers to the rotation on the support points to be equal to zero. The second one refers to the rotation on the left and right side of spiral spring to be equal. The third is a natural geometry condition to be in its equilibrium on spiral spring point.

β1= 0 at z1 = 0 and β2 = 0 at z2 = L/2

βL = βR at z1 = L/2 and z2 = 0

GJ.β1' = GJ.β2' – C.β2 at z1 = L/2 and z2 = 0 ....(5)

By substituting all boundary conditions in Equations 5 to Equations 4, all the constants of integration can be obtained as follows:

$$B1 = 0$$

$$A2.\sin(kL/2) + B2.\cos(kL/2) = 0$$

$$A1.\sin(kL/2) + B2.\sin(kL/2) - B2 = 0$$

$$A1.k\cos(kL/2) - B1.k\sin(kL/2) - A2.k + B2.C = 0 \qquad (6)$$

In a matrix form, Equation 6 can be rewritten as,

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & SL_2 & CL_2 \\ SL_1 & CL_1 & 0 & -1 \\ kCL_1 & -kSL_1 & -k & S \end{pmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (7)$$

Notation SL_1 refers to mathematical eq. $\sin(kL_1)$ with L_1 equals to L/2, and similarly for SL_2. The other notation, CL_1, refers to $\cos(kL_1)$ with L_1 equals to L/2 and similarly for CL_2. According to Eq. 1, variables in the first matrix form contain k variable which is derived from the critical moment. The critical moment can therefore be obtained by setting a definite value of moment that provide a zero determinant value of the first matrix form. In this case, the critical moment cannot be determined directly. A trial and error method should be used by setting the determinant to be equal to zero.

In a case whereby the beam is handled by several spiral springs, Eq. 4 can be expanded correspondingly. Assuming the spiral springs are in n numbers, the solution of differential equation will be (n+1) numbers. Thus, there will be 2(n+1) constants of integration to be determined. By applying boundary and natural geometry conditions as mentioned in Eq. 5, 2(n+1) constant of integrations of the homogeneous equations will be derived. By trial and error method, the critical moment can be, as well, defined by setting the determinant of coefisien matrix to be equal to zero.

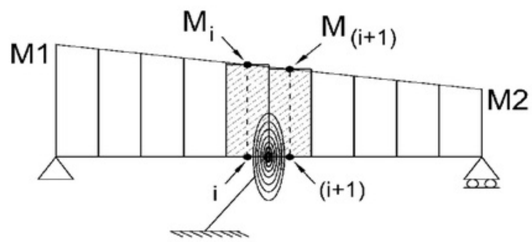

Figure 3. Moment segmentation diagram

If the momen is not a constant value, the moment shall be divided into several segments. Each segment has a constant moment equal to the moment of the middle segment, an average value of M1 and M2. In the same manner, the solution of differential equation of each segment can be derived. Each segment has 2 constant of integration. If the beam is divided into n segments, the number of constant of integration equals to 2n. Making use of Eq. 5 for z equals to the position of joint between two adjacent segments, 2n homogenous equations of the constant of integrations can then be derived. If the joint has no spiral spring, the C value in Eq. 5 has to be zero. By trial and error, the critical moment can be, as well, defined. It is a first mode critical moment, that if it is substituted to the homogeneous equation of the constant of integration, the first mode shape of buckling can be drawn.

3. RESULTS

Critical Moment

A simply supported beam with L length is subjected to a constant moment Mx. The spiral springs are added into the beam system. The stiffness of such spring are varied from C equals to 0 to 1. To study the effects of spiral springs, 0 to 5 spiral springs are added to the beam. Such springs divide the beam into the same segment's length. The result is shown in Figure 4.

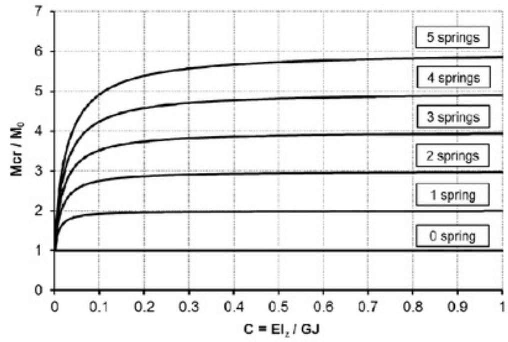

Figure 4. Ratio between Mcr to M_0 wrt. the number of spiral springs with C = 0 - 1

According to Figure 4, the beam's critical moment (M_{cr}) having n number of spiral springs with C equals to 1 will approach (n+1) times its critical moment without spiral spring (M_0). The curve shows a comparation value.

Mode shape

A simply supported beam having L length and one spiral spring in its midlength is subjected to a constant moment Mx. The spiral spring stiffness varies from 0 to 1. The beam's first mode shape is shown in Figure 5.

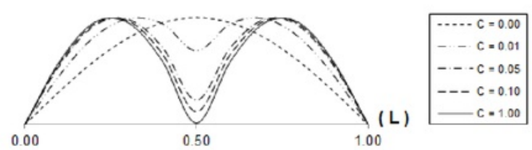

Figure 5. The first mode shape of beam with one spiral spring

As can be seen in Figure 5, the mode shape of the beam slightly changes as the spiral spring's stiffness increase. Lastly, the mode shape finds its steady state when the spiral spring stiffness equals to 1 (C = 1).

Similarly, the first mode shape of beam with two spiral springs of various stiffness (C from 0 to 1) are calculated and plotted as shown in Figure 6. The results are of similar agreement as those in Figure 5 that the mode shape of the beam slightly changes as the spiral spring's stiffness increase.

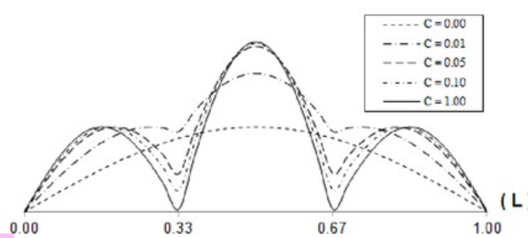

Figure 6. The first mode shape of beam with two spiral springs

Beam's critical moment of linear moment distribution

In this case the beam is assumed to have a linear moment distribution as shown in Figure 3, the aforementioned equations as given in Section 2 can also be used to solve this case. By discretizing the moment field into small piece of segments by assuming each piece of moment field as a constant moment, the critical moment can be obtained when the determinant of matrix [M] equals to zero.

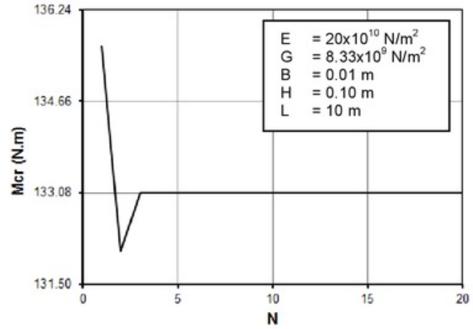

Figure 7. Beam's critical moment for M2/M1 = 0.5, no spring

The critical moment for this case is plotted as in Figure 7. The results show that as the number of segments (N) increases, the moment obtained will converge to the real critical moment M_{cr} that in this case nearly reaches 133.08 N.m.

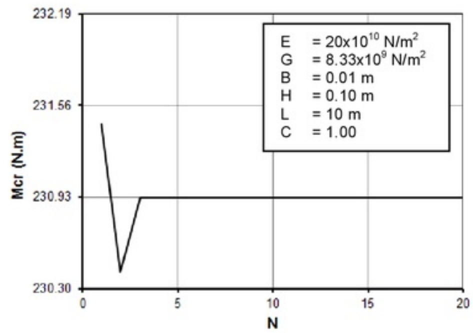

Figure 8. Beam's critical moment for M2/M1 = 0.5, 1 spring on beam midlength

The convergence of the critical moment also occurs for a trapezoidal moment distribution as shown in Figure 8. In this case, the critical moment M_{cr} is around 230.93 N.m.

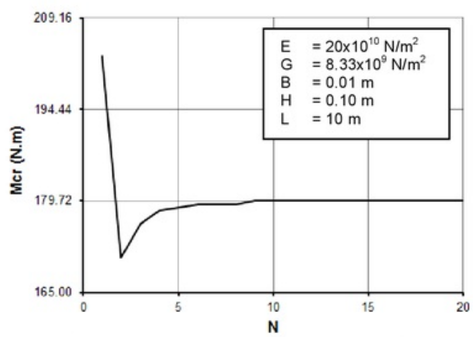

Figure 9. Beam's critical moment for M2/M1 = 0, no spring

The convergence of the critical moment also occurs for a trapezoidal moment distribution as shown in Figure 9. In this case, the critical moment M_{cr} is around 179.72 N.m.

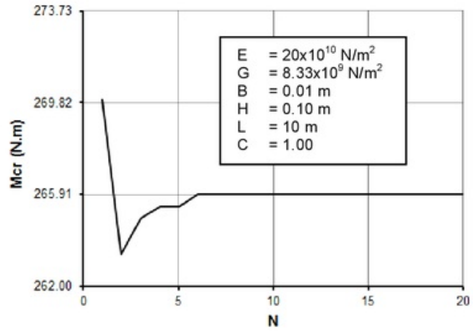

Figure 10. Beam's critical moment for M2/M1 = 0, 1 spring on beam midlength

The convergence of the critical moment also occurs for a trapezoidal moment distribution as shown in Figure 10. In this case, the critical moment M_{cr} is around 265.91 N.m.

4. CONCLUSION

Based on the analyses result, three main conclusions are drawn as follows:

1. The presence of spiral springs with certain stiffness will improve critical moment of the beam.
2. The critical moment of beam having n numbers of spiral springs with stiffness C = 1 will approach (n+1) times of the critical moment of beam without spring.
3. By increasing spiral spring's stiffness, rotational displacements on spring joints on the beam are almost equal to zero.
4. The critical moment of the beam with non-constant moment distribution will converge to a value by discretizing the beam into more segments.

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